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Condensate Enhancement and D-Meson Mixing in Technicolor Theories

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Since the pioneering work of Eichten and Lane it has been known that the scale of the interactions responsible for the generation of the strange-quark mass in extended technicolor theories must, absent any "GIM-like" mechanism for suppressing flavor-changing neutral currents, be greater than of order 1000 TeV. In this note we point out that the constraint from the neutral D-meson system is now equally strong, implying that the charm quark mass must also arise from flavor dynamics at a scale this high. We then quantify the degree to which the technicolor condensate must be enhanced in order to yield the observed quark masses, if the extended technicolor scale is of order 1000 TeV. Our results are intended to provide a framework in which to interpret and apply the results of lattice studies of conformal strongly interacting gauge theories, and the corresponding numerical measurements of the anomalous dimension of the mass operator in candidate theories of "walking" technicolor.

I. INTRODUCTION

Technicolor [1–3] provides a dynamical mechanism for electroweak symmetry breaking in which the weak interactions are spontaneously broken to electromagnetism via technifermion chiral symmetry breaking (which is analogous to quark chiral symmetry breaking in QCD). While technicolor chiral symmetry breaking alone is sufficient to generate the masses of the weak gauge bosons, additional "extended technicolor" (ETC) interactions [4, 5] are required to couple the symmetry breaking sector to the quarks and leptons and thereby generate ordinary fermion masses. As noted by Eichten and Lane [5], however, the additional interactions introduced to generate ordinary fermion masses cannot be flavor-universal, and would therefore also give rise to flavor-changing neutral-current (FCNC) processes. In particular they showed that, absent any "GIM-like" mechanism [6–9] for suppressing flavor-changing neutral currents, the ETC scale associated with strange-quark mass generation must be larger than of order 10^3 TeV in order to avoid unacceptably large (CP-conserving) contributions to neutral K-meson mixing. To obtain quark masses that are large enough therefore requires an enhancement of the technifermion condensate over that expected naively by scaling from QCD. Such an enhancement can occur in "walking" technicolor theories in which the gauge coupling runs very slowly [10–15][1], or in "strong-ETC" theories in which the ETC interactions themselves are strong enough to help drive technifermion chiral symmetry breaking [18–22].[2]

In this paper we update the bounds on ETC interactions derived from limits on flavor-changing neutral-currents. In particular we show that the bound on the scale of ETC interactions arising from D-meson mixing is now as constraining as that arising from CP-conserving contributions to K-meson mixing and, therefore, absent any mechanism for the suppression of flavor-changing neutral-currents [6–9], the ETC scale associated with *charm*-quark mass generation must also be larger than of order 10^3 TeV. Since the charm quark is so much heavier than the strange quark, requiring an ETC model to produce m_c from interactions at a scale of over 1000 TeV is a significantly stronger

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[1] For some examples of proposed models of walking technicolor, see [16] and [17] and references therein.

[2] It is also notable that walking technicolor and strong-ETC theories are quite different from QCD, and may be far less constrained by precision electroweak measurements [3, 23, 24].

constraint on model-building than the requirement of producing m_s at that scale. Subsequently, we quantify the amount of technicolor condensate enhancement required to produce a given quark mass if the ETC scale is of order 10^3 TeV. Our quantitative results are intended to provide a framework within which to interpret and apply lattice Monte Carlo studies of candidate walking technicolor theories, such as those in Refs. [25–36].

II. CONSTRAINTS ON Λ_{ETC} FROM NEUTRAL MESON MIXING

At low energies, the flavor-changing four-fermion interactions induced by ETC boson exchange alter the predicted rate of neutral meson mixing. Ref. [37] has derived constraints on general $\Delta F = 2$ four-fermion operators that affect neutral Kaon, D-meson, and B-meson mixing, including the effects of running from the new physics scale down to the meson scale and interpolating between quark and meson degrees of freedom. Their limits on the coefficients (C^1_j) of the FCNC operators involving LH current-current interactions:

$$C^1_K(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L) \tag{1}$$
$$C^1_D\,(\bar{c}_L\gamma^\mu u_L)(\bar{c}_L\gamma_\mu u_L) \tag{2}$$
$$C^1_{B_d}(\bar{b}_L\gamma^\mu d_L)(\bar{b}_L\gamma_\mu d_L) \tag{3}$$
$$C^1_{B_s}(\bar{b}_L\gamma^\mu s_L)(\bar{b}_L\gamma_\mu s_L)\,, \tag{4}$$

are listed in the left column of Table I. In their notation, the generic form of the coefficient C^1_i is:

$$C^1_i = \frac{F_i L_i}{\Lambda^2} \tag{5}$$

where F_i is a flavor factor that is expected to be $|F_i| \sim 1$ in a model with an arbitrary flavor structure and absent any GIM-like mechanism [6–9] for suppressing flavor-changing neutral currents; L_i is a loop factor that is simply 1 in a model with tree-level FCNC; and Λ is the scale of new physics.

TABLE I: Limits from the UTFit Collaboration [37] on coefficients of left-handed four-fermion operators contributing to neutral meson mixing (left column) and the implied lower bound on the ETC scale (right column). The bounds in the first four rows apply when one assumes ETC does not contribute to CP violation; the bound in the last row applies if one assumes that ETC does contribute to CP violation in the Kaon system.

Bound on operator coefficient (GeV^{-2})	Implied lower limit on ETC scale (10^3 TeV)
$-9.6\times10^{-13} < \Re(C^1_K) < 9.6\times10^{-13}$	1.0
$\lvert C^1_D\rvert < 7.2\times10^{-13}$	1.5
$\lvert C^1_{B_d}\rvert < 2.3\times10^{-11}$	0.21
$\lvert C^1_{B_s}\rvert < 1.1\times10^{-9}$	0.03
$-4.4\times10^{-15} < \Im(C^1_K) < 2.8\times10^{-15}$	10

In the case of an ETC model with arbitrary flavor structure and no assumed ETC contribution to CP-violation, one has $C^1_i = \Lambda^{-2}_{ETC}$ and the limits on the Λ_{ETC} from [37] are as shown in the right-hand column of Table I. The lower bound on Λ_{ETC} from D-meson mixing is now the strongest, with that from Kaon mixing a close second and those from B-meson mixing far weaker. Since the charm quark is so much heavier than the strange quark, requiring an ETC model to produce m_c from interactions at a scale of over 1000 TeV is a significantly stronger constraint on model-building than the requirement of producing m_s at that scale. Note that if one, instead, assumes that ETC contributes to CP-violation in the Kaon system, then the relevant bound on Λ_{ETC} comes from the imaginary part of C_K^1 and is a factor of ten more severe (see last row of Table I).

In the next section we will quantify the amount of technicolor condensate enhancement required to produce a given quark mass if the ETC scale is of order 10^3 TeV.

III. CONDENSATE ENHANCEMENT AND γ_m

In studying how ETC theories produce quark masses, the primary operator of interest has the form[3]

$$\frac{(\bar{Q}^a_L \gamma^\mu q^j_L)(u^i_R \gamma_\mu U^a_R)}{\Lambda^2_{ETC}} \,, \tag{6}$$

where the Q^a_L and U^a_R are technifermions (a is a technicolor index), and the q^j_L and u^i_R are left-handed quark doublet and right-handed up-quark gauge-eigenstate fields (i and j are family indices). This operator will give rise, after technifermion chiral symmetry breaking at the weak scale, to a fermion mass term of order

$$\mathcal{M}_{ij} = \frac{\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}}}{\Lambda^2_{ETC}} \,. \tag{7}$$

Here it is important to note that the technifermion condensate, $\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}}$ is renormalized at the ETC scale [10–15]. It is related to the condensate at the technicolor (electroweak symmetry breaking) scale by

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}} = \exp\left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \gamma_m(\alpha_{TC}(\mu)) \frac{d\mu}{\mu} \right) \langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \,, \tag{8}$$

where $\gamma_m(\alpha_{TC}(\mu))$ is the anomalous dimension of the technifermion mass operator.[4] Using an estimate of the technifermion condensate, and a calculation of the anomalous dimension of the mass operator, we may estimate the size of quark mass which can arise in a technicolor theory for a given ETC scale.

In a theory of walking technicolor [10–15], the gauge coupling runs very slowly just above the technicolor scale Λ_{TC}. The largest enhancement occurs in the limit of "extreme walking" in which the technicolor coupling, and hence the anomalous dimension γ_m, remains approximately constant from the technicolor scale, Λ_{TC}, all the way to the ETC scale, Λ_{ETC}. In the limit of extreme walking, one obtains

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{ETC}} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma_m} \langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \,. \tag{9}$$

We may now use (9) to quantify the enhancement of the technicolor condensate required to produce the observed quark masses in a walking model. Specifically, we will investigate the size of the quark mass which can be achieved in the limit of extreme walking for various γ_m, and an ETC scale of 10^3 TeV (which, as shown above, should suffice to meet the CP-conserving FCNC constraints in the K - and D-meson systems). The calculation requires an estimate of the technicolor scale Λ_{TC} and the technicolor condensate renormalized at the electroweak scale, $\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}}$.

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[3] In an ETC gauge theory, we would expect $1/\Lambda^2_{ETC} \equiv g^2_{ETC}/M^2_{ETC}$ where g_{ETC} and M_{ETC} are the appropriate extended technicolor coupling and gauge-boson mass, respectively. At energies below M_{ETC}, these parameters always appear (to leading order in the ETC interactions) in this ratio – and therefore, we use Λ_{ETC} for simplicity.

[4] We will address the potential scheme-dependence of γ_m below.

Two estimates of the scales associated with technicolor chiral symmetry breaking are commonly used in the literature: Naive Dimensional Analysis (NDA) [38–40] and simple dimensional analysis (DA) as applied in [5]. In Naive Dimensional Analysis, one associates Λ_{TC} with the "chiral symmetry breaking scale" for the technicolor theory, and hence $\Lambda_{TC} = \Lambda_{\chi SB} \approx 4\pi v$ (where $v \approx 250$ GeV is the analog of f_π in QCD), while

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \approx \frac{\Lambda^3_{\chi SB}}{(4\pi)^2} \ . \tag{10}$$

Inserting these relations into eqns. (7) and (8) we find, in the limit of extreme walking (constant γ_m)

$$\begin{aligned} m_q^{NDA} &= \frac{\Lambda_{\chi SB}}{(4\pi)^2} \left(\frac{\Lambda_{\chi SB}}{\Lambda_{ETC}} \right)^{2-\gamma_m} && (11) \\ &\approx 19.9\,\mathrm{GeV} \cdot (3.14 \times 10^3)^{2-\gamma_m} \ , && (12) \end{aligned}$$

where the last equality applies for an ETC scale of 10^3 TeV. Alternatively, in the simple dimensional estimates given for example in [5], one simply assumes that all technicolor scales are given by $\Lambda_{TC} \approx 1$ TeV, and hence one uses

$$\langle \bar{U}_L U_R \rangle_{\Lambda_{TC}} \approx \Lambda^3_{TC}. \tag{13}$$

In this case, one finds

$$\begin{aligned} m_q^{DA} &= \Lambda_{TC} \left(\frac{\Lambda_{TC}}{\Lambda_{ETC}} \right)^{2-\gamma_m} && (14) \\ &\approx 1000\,\mathrm{GeV} \cdot \left(1.0 \times 10^{-3}\right)^{2-\gamma_m} \ , && (15) \end{aligned}$$

where, again, the last equality applies for an ETC scale of 10^3 TeV. Note that in neither case have we included factors to correct for the number of weak doublets in the technicolor sector, nor attempted to account for the "large-N_{TC}" limit [3] – however, such factors can only *suppress* the size of the quark masses generated.

In Table II we use eqns. (12) and (15) to estimate the size of quark mass corresponding to various (constant) values of γ_m and an ETC scale of 10^3 TeV. We show these values in the range $0 \leq \gamma_m \leq 2.0$ since $\gamma_m \simeq 0$ in a "running" technicolor theory, and conformal group representation unitarity implies that $\gamma_m \leq 2.0$ [41]. The usual Schwinger-Dyson analysis used to analyze technicolor theories would imply that $\gamma_m \leq 1.0$ in walking technicolor theories [10–15], while the values $1.0 \leq \gamma_m \leq 2.0$ could occur in strong-ETC theories [18–22].

TABLE II: Size of the quark mass m_q generated by technicolor dynamics assuming an ETC scale $\Lambda_{ETC} = 1000$ TeV and various values for the anomalous dimension γ_m of the mass operator. In the row labeled NDA [DA], the value of the techniquark condensate at the technicolor scale is taken to be $\langle \bar{T}T \rangle \approx (580\,\mathrm{GeV})^3$ $[(1000\,\mathrm{GeV})^3]$. Values of γ_m of 1.0 or less correspond to walking theories [10–15]; values greater than 1.0 correspond to strong-ETC theories [18–22].

γ_m	0	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0
m_q^{NDA}	0.2 MeV	0.8 MeV	3.5 MeV	15 MeV	63 MeV	260 MeV	1.1 GeV	4.7 GeV	20 GeV
m_q^{DA}	1 MeV	5.6 MeV	32 MeV	180 MeV	1 GeV	5.6 GeV	32 GeV	180 GeV	1 TeV

IV. DISCUSSION

Examining Table II, we see that generating the charm quark mass from ETC dynamics at a scale of order 10^3 TeV requires an anomalous dimension γ_m close to or exceeding one, even in the case of the more generous DA estimate of the technifermion condensate. It will therefore be helpful for nonpertubative studies of strong technicolor dynamics to determine how large γ_m can be in specific candidate theories of walking technicolor. Values of γ_m substantially less than one would require a lower ETC scale, which would necessitate the construction of ETC theories with approximate flavor symmetries [6–9] and corresponding GIM-like partial cancellations of flavor-changing contributions. Note also that our quark mass estimates are generous on several fronts: taking into account the number of weak doublets in the technicolor sector, large N_{TC} effects, or less extreme walking of the technicolor coupling would suppress the size of the quark mass generated.

Our results further suggest that the nonperturbative study of strong-ETC models [18–22] may also be useful, since generating the heavy quark masses may be easier in such models. In this case, it will be particularly interesting to see if such theories contain a light, but broad, scalar Higgs-like resonance and whether they could avoid potentially dangerous custodial symmetry violating contributions to M_W^2/M_Z^2 [42–44].

Finally we should address a subtlety in our discussion: γ_m is only scheme-independent at an IR fixed point where the gauge theory is conformal. In fact, lattice Monte Carlo studies to date [25–36] focus on establishing the "conformal window" of strongly coupled theories within which, since the theory is truly conformal, chiral symmetry breaking (and therefore electroweak symmetry breaking) would not occur. As we discuss above, however, candidate walking-technicolor theories will likely be very close to conformal over a large range of energy scales – namely, they are expected to be approximately conformal over the three orders of magnitude that separate the technicolor and ETC scales. Therefore, measurements of γ_m in the conformal phase of these theories can suggest (via the results of Table II) which models might form the basis of a realistic technicolor model, when either "tuned" to be slightly away from the fixed point trajectory, or deformed [45] by the presence of some additional operator (see also the discussion of the utility of working in the conformal phase in Ref. [32]). Ultimately, it would be desirable to simulate a nearly conformal walking-technicolor theory in the phase of broken chiral symmetry – in which case the relevant technicolor condensate and ETC-generated ordinary fermion masses can be measured directly.

In this paper we have noted that constraints on FCNC in the neutral D-meson system imply that the charm quark mass must, like the strange quark mass, arise from flavor dynamics at a scale of order 10^3 TeV. We have also quantified the degree to which the technicolor condensate must be enhanced in order to yield the observed quark masses, if the extended technicolor scale is of order 10^3 TeV. Our results provide a framework in which to interpret and apply the results of lattice studies of conformal strongly interacting gauge theories, and the corresponding numerical measurements of the anomalous dimension of the mass operator in candidate theories of walking technicolor.

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